Properties of Stars

1.1 Brightnesses of Stars

I'm sure you have noticed that some stars are very bright, others less bright, and some are quite dim. There is a "true" brightness of a star and an "apparent" brightness of a star. A truly bright star may appear to be quite faint if it is far away from the person observing it. When you look up at the night sky you observe the "apparent" brightnesses of stars. We are interested in the true brightness because this gives us information on the inner workings of a star. More on that in later chapters!

1.1.1 Luminosity

Luminosity is a measure of the true brightness of a star. How luminous a star is (L) depends on the size of the star (radius R) and its surface temperature (T). Luminosity is energy radiated per second (the units are Watts $\rightarrow$ W).

$$L = 4\pi \, R^2 \, \sigma \, T^4$$

Example

What is the luminosity of a star if its surface temperature is 7000 K and its radius is 5.00 x 10^9 m?

$$\begin{aligned} L &= 4\pi \, R^2 \, \sigma \, T^4 \\ L &= 4\pi \, (5.00 \, \mathrm{x} \, 10^9)^2 \, (5.67 \, \mathrm{x} \, 10^{-8}) \, (7000)^4 \\ L &= 4\pi \, (2.50 \, \mathrm{x} \, 10^{19}) \, (5.67 \, \mathrm{x} \, 10^{-8}) \, (2.40 \, \mathrm{x} \, 10^{15}) \\ L &= 4.28 \, \mathrm{x} \, 10^{28} \, \mathrm{W} \end{aligned}$$

Wow! That is a *big* number! It might be helpful to compare this brightness to the brightness of the Sun.

$$L_{Sun} = 3.83 \,\mathrm{x}\, 10^{26}\,\mathrm{W}$$

So lets compare to the Sun ...

$$\frac{L}{L_{Sun}} = \frac{4.28 \,\mathrm{x}\, 10^{28}}{3.83 \,\mathrm{x}\, 10^{26}} = 112$$

Okay$\cdots$ this means that the star is 112 times brighter than the Sun. This can be written as $L = 112\ L_{Sun}$.

<u>Notation Alert!</u> Note that most textbooks use the symbol $L_{\odot}$ for L_{Sun} and the ratio we calculated above $\frac{L}{L_{Sun}}$ is often written in a more compact way as $\mathcal{L} = \frac{L}{L_{Sun}}$. You have been warned!

1.1.2 Apparent Brightness

How bright a star appears to be B (its "apparent" brightness) depends on the "true" brightness of the star L and the distance to the star d.

$$B = \frac{L}{4\pi d^2}$$

<u>Example</u>
How bright does the star mentioned above appear to be at a distance of 15 Ly? (1 Ly = 9.46 x 10^{15} m)

$$\begin{aligned} B &= \frac{L}{4\pi d^2} \\ B &= \frac{4.28 \,\mathrm{x}\, 10^{28}}{4\pi\,(1.42 \,\mathrm{x}\, 10^{17})^2} \\ B &= 1.69 \,\mathrm{x}\, 10^{-7}\,\mathrm{W/m^2} \end{aligned}$$

1.1.3 The Magnitude Scale

Luminosity is not the only way to describe the brightnesses of stars. The magnitude scale originated with Hipparchus around 150 B.C. This scale is still used today - notably on nearly all star charts. Large positive magnitudes indicate very faint stars, whereas magnitudes near zero (or even negative) indicate bright stars. Just like luminosity, there are "true" magnitudes (M_v) and "apparent" magnitudes (m_v). True magnitudes are also referred to as "absolute" magnitudes.

The relationship between luminosity and absolute magnitude is

$$\frac{L}{L_{Sun}} = 10^{-\frac{2}{5}\,(M_v - M_{v\,Sun})}$$

With a little bit of algebra this can also be written as

$$\log\left(\frac{L}{L_{Sun}}\right) = -\frac{2}{5}\left(M_v - M_{v\,Sun}\right)$$

<u>Example</u>

What is the luminosity of a star if its absolute magnitude is 2.35?
(Note: $M_{v\,Sun} = 4.83$, and $L_{Sun} = 3.83 \text{ x } 10^{26}$ W)

$$\begin{aligned}
\frac{L}{L_{Sun}} &= 10^{-\frac{2}{5}\,(M_v - M_{v\,Sun})} \\
\frac{L}{L_{Sun}} &= 10^{-\frac{2}{5}\,(2.35-4.83)} \\
\frac{L}{L_{Sun}} &= 10^{0.992} \\
\frac{L}{L_{Sun}} &= 9.82
\end{aligned}$$

Therefore

$$L = 9.82\,L_{Sun} = 3.76\,\text{x}\,10^{27}\,\text{W}$$

Example

What is the absolute magnitude of a star if its luminosity is 150 times the luminosity of the sun ($L = 150\ L_{Sun}$)?

$$\begin{aligned}
\log\left(\frac{L}{L_{Sun}}\right) &= -\frac{2}{5}\left(M_v - M_{v\,Sun}\right) \\
-\frac{2}{5}\left(M_v - M_{v\,Sun}\right) &= \log\left(\frac{L}{L_{Sun}}\right) \\
-\frac{2}{5}\left(M_v - 4.83\right) &= \log(150) \\
M_v - 4.83 &= -\frac{5}{2}\log(150) \\
M_v - 4.83 &= -5.44 \\
M_v &= -0.61
\end{aligned}$$

1.2 Calculating the Size of a Star

Stars are so far away that it is not possible to directly measure their sizes. Knowing the luminosity and surface temperature of a star allows us to calculate its radius (R). Lets see how...

The luminosity of any star is given by

$$L = 4\pi\, R^2\, \sigma\, T^4$$

The luminosity of the Sun is given by

$$L_{Sun} = 4\pi\, R_{Sun}^2\, \sigma\, T_{Sun}^4$$

Now lets do a little algebra!

$$\frac{L}{L_{Sun}} = \frac{4\pi\, R^2\, \sigma\, T^4}{4\pi\, R_{Sun}^2\, \sigma\, T_{Sun}^4} = \frac{R^2\, T^4}{R_{Sun}^2\, T_{Sun}^4}$$

$$\frac{R^2}{R_{Sun}^2} = \frac{L\, T_{Sun}^4}{L_{Sun}\, T^4} = \left(\frac{T_{Sun}}{T}\right)^4 \left(\frac{L}{L_{Sun}}\right)$$

$$\frac{R}{R_{Sun}} = \left(\frac{T_{Sun}}{T}\right)^2 \sqrt{\frac{L}{L_{Sun}}}$$

<u>Example</u>
What is the radius of a star if it is 25 times brighter than the Sun and has a surface temperature of 8000 K? (Note: Surface temperature of the Sun is 5780 K)

$$\frac{R}{R_{Sun}} = \left(\frac{T_{Sun}}{T}\right)^2 \sqrt{\frac{L}{L_{Sun}}} = \left(\frac{5780}{8000}\right)^2 \sqrt{25} = 2.6$$

So this star is 2.6 times larger than the Sun. The radius is

$$R = 2.6\, R_{Sun} = 1.8 \,\mathrm{x}\, 10^9\,\mathrm{m}$$

<u>Notation Alert!</u> Note that most textbooks use the symbol $R_\odot$ for R_{Sun}.

1.3 Calculating Distance

Knowing the distance to a star is necessary to calculate the luminosity of a star (first measure the apparent brightness and the distance). Here we discuss two methods used to determine distance.

1.3.1 The Parallax Method

This method is very simple to use. Given the measured parallax, the distance in parsecs (pc) is easily calculated. (Note that 1 pc = 3.26 Ly)

$$d_{pc} = \frac{1}{p}$$

<u>Example</u>
What is the distance to a star if it has a parallax of 0.0150 arc-seconds?

$$d_{pc} = \frac{1}{p} = \frac{1}{0.015} = 66.7\,\mathrm{pc}$$

The distance to this star in light-years is

$$d_{Ly} = 66.7\,(3.26) = 217\,\mathrm{Ly}$$

1.3.2 Standard Candle Method

Unfortunately, the parallax method only works for stars less than 500 pc away (the angles become too small to accurately measure). Another method that is used is the Standard Candle Method.

The idea is as follows: if you know the luminosity L of an object (variable star, supernova, etc.), and measure the apparent brightness B of the object, you can calculate the distance.

$$\begin{aligned} B &= \frac{L}{4\pi d^2} \\ d^2 &= \frac{L}{4\pi B} \\ d &= \sqrt{\frac{L}{4\pi B}} \end{aligned}$$

Example

Find the distance to a star if its luminosity is known to be 3.00 x 10^{33} W, and its apparent brightness is measured to be 0.00000320 W/m^2?

$$d = \sqrt{\frac{L}{4\pi B}} = \sqrt{\frac{3.00 \,\mathrm{x}\, 10^{33}}{4\pi\,(0.00000320)}} = 8.64 \,\mathrm{x}\, 10^{18}\,\mathrm{m} \;= 913\,\mathrm{Ly}$$

1.4 Stellar Masses

The mass of a star is perhaps its most important property. As we will find out in later chapters, the mass of a star determines how a star lives, how long it will live, and how it will die.

1.4.1 Newton's Method

Newton found a method to determine the masses of objects that orbit each other. The formula is found below and is a modified version of Kepler's third law.

$$M_1 + M_2 = \frac{a^3}{P^2}$$

Example

What is the mass of a super giant star if it is orbited by a star that has 3 times the mass of the Sun, takes 2 years to complete one orbit, and averages 5 AU away?

$$\begin{aligned} M_1 + M_2 &= \frac{a^3}{P^2} \\ M_{SG} + 3.00 &= \frac{(5.00)^3}{(2.00)^2} \\ M_{SG} + 3.00 &= 31.25 \\ M_{SG} &= 28.25\, M_{Sun} \\ M_{SG} &= 5.62 \,\mathrm{x}\, 10^{31}\,\mathrm{kg} \end{aligned}$$

1.4.2 Main Sequence Stars: Luminosity and Mass

There is a special relationship between luminosity and mass for Main Sequence stars. This relationship holds only for Main Sequence stars!

If we define a star's mass in terms of the Sun's mass

$$\mathcal{M} = \frac{M}{M_{Sun}}$$

And define a star's luminosity in terms of the Sun's luminosity

$$\mathcal{L} = \frac{L}{L_{Sun}}$$

Then the following relationship holds for Main Sequence stars

$$\mathcal{M}^3 = \mathcal{L}$$

Therefore

$$\mathcal{M} = (\mathcal{L})^{\frac{1}{3}}$$

Example

What is the mass of a Main Sequence star if it is 18 times brighter than the Sun?

$$\mathcal{M} = (\mathcal{L})^{\frac{1}{3}} = (18)^{\frac{1}{3}} = 2.62$$

Since

$$\mathcal{M} = \frac{M}{M_{Sun}}$$

This implies this Main Sequence star has a mass

$$M = 2.62\, M_{Sun} = 5.24 \,\mathrm{x}\, 10^{30}\,\mathrm{kg}$$