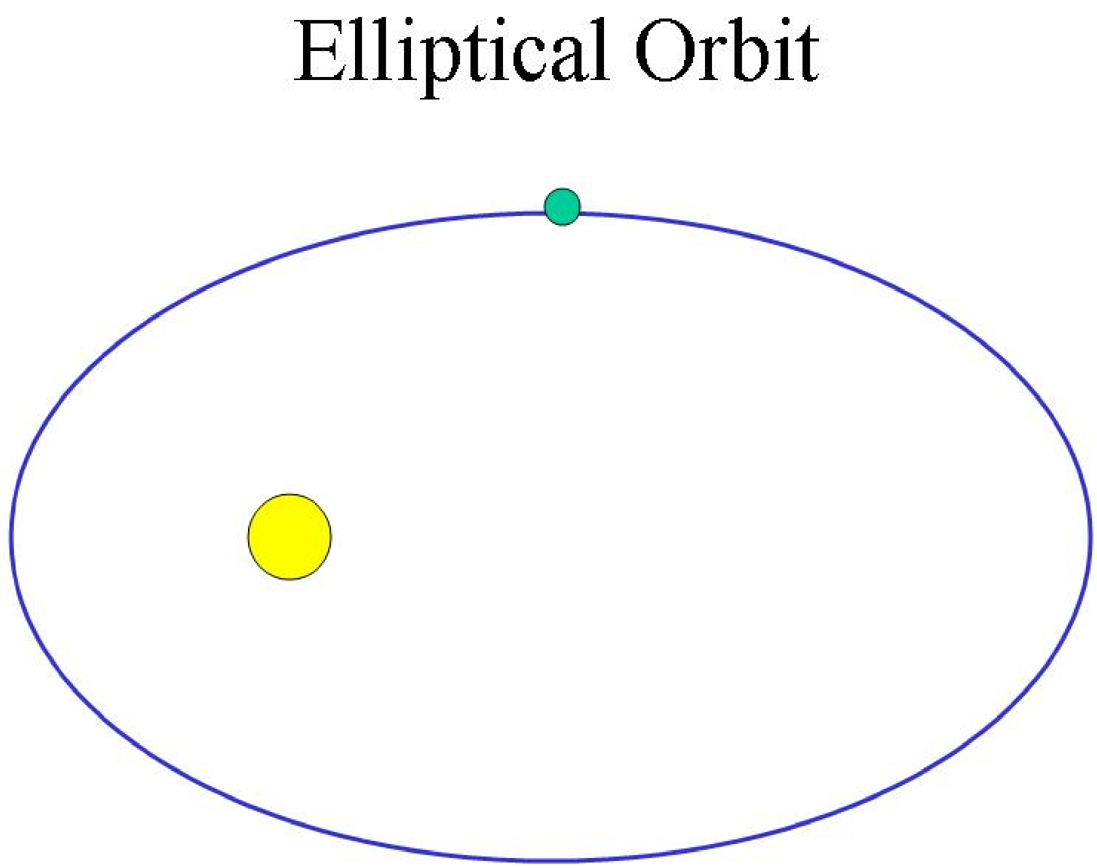
Orbits and Gravity

1.1 Dealing with Kepler's 3$^{\text{rd}}$ Law: $P^2 = a^3$

Kepler's three laws of planetary motion deal with objects that orbit the Sun in elliptical orbits. Kepler's third law deals with the relationship between the average orbital distance a from the Sun (in astronomical units AU) and the period of time needed for one orbit (in years).



Two types of problems arise that require the use of the third law:

Given the average distance a, calculate the time for one orbit P

Given the time for one orbit P, calculate the average distance a.

Let's take a look at some examples below...

1.1.1 Finding the Period P

Example

How much time does it take a spacecraft to orbit the Sun if its average distance from the Sun is 8.00 AU?

$$\begin{aligned} P^2 &= a^3 \\ P^2 &= (8.00)^3 \\ P^2 &= 512 \\ P &= \sqrt{512} \\ P &= 22.6 \ years \end{aligned}$$

Example

How much time does it take a comet to orbit the Sun if its closest distance from the Sun is 0.38 AU and its greatest distance from the Sun is 7.44 AU?

First find a (the average distance)

$$\begin{aligned} a &= \frac{d_1 + d_2}{2} \\ a &= \frac{0.38 + 7.44}{2} \\ a &= \frac{7.82}{2} \\ a &= 3.91 \ AU \end{aligned}$$

Second find P

$$\begin{aligned} P^2 &= a^3 \\ P^2 &= (3.91)^3 \\ P^2 &= 59.77\cdots \\ P &= \sqrt{59.77\cdots} \\ P &= 7.73\ years \end{aligned}$$

1.1.2 Finding the average distance a

Example

Bob's asteroid takes 12.0 years to orbit the Sun. What is the average distance of Bob's asteroid?

$$\begin{aligned} P^2 &= a^3 \\ a^3 &= P^2 \\ a^3 &= (12.0)^2 \\ a^3 &= 144 \\ a &= \sqrt[3]{144} = (144)^{\frac{1}{3}} \\ a &= 5.24\ AU \end{aligned}$$

1.2 Newton's Law of Gravity

Example

What is the attractive force between two asteroids that are 50.00 *km* apart if the mass of the first asteroid is known to be 5.920 x 10^{15} *kg* and the other is 2.320 x 10^{10} *kg*?

$$
\begin{aligned}
F &= \frac{G\,M_1\,M_2}{d^2} \\
F &= \frac{(6.673 \times 10^{-11})\,(5.920 \times 10^{15})\,(2.320 \times 10^{10})}{(50,000)^2} \\
F &= \frac{(6.673 \times 10^{-11})\,(5.920 \times 10^{15})\,(2.320 \times 10^{10})}{2.500 \times 10^9} \\
F &= \frac{9.165 \times 10^{15}}{2.500 \times 10^9} = 3.666 \times 10^6\,N
\end{aligned}
$$

Notice that we *had* to convert the distance from kilometers to meters (50 *km* = 50,000 *m*) to get the correct answer using this formula.

1.3 Escape Velocity

Example

At what speed must a rocket be moving to escape a comet that has a mass of 3.000 x 10^{20} *kg* and a radius of 15.00 *km*?

$$
\begin{aligned}
V_{escape} &= \sqrt{\frac{2\,GM}{R}} \\
V_{escape} &= \sqrt{\frac{2\,(6.673\,\mathrm{x}\,10^{-11})(3.000\,\mathrm{x}\,10^{20})}{15,000}} \\
V_{escape} &= \sqrt{\frac{4.0038\,\mathrm{x}\,10^{10}}{15,000}} \\
V_{escape} &= \sqrt{2,669,199.99} = 1,634\,m/s
\end{aligned}
$$

Notice that we *had* to convert the radius from kilometers to meters (15 km = 15,000 m) to get the correct answer using this formula. To get a feeling for how fast 1634 m/s is you could compare it to the speed of sound (330 m/s) or multiply by 2.2 to get MPH. So the escape velocity is 1634 m/s which is around Mach 5 or around 3595 MPH. We will not be using MPH or the Mach number for homework or exams… I merely show it here to give something to compare to.