Density and Blackholes

The different types of stars (supergiant, giant, main sequence, and white dwarf) vary greatly in terms of size and mass. Density is a measure of how tightly matter is packed together. Density also varies greatly among the types of stars. Black Holes formally have *infinite* density!

1.1 Density

Density is the amount of mass per unit of volume. Typically, density is measured in grams per cubic centimenter (g/cm^3). For comparison, the density of water is 1.00 g/cm^3 and the density of gold is 19.30 g/cm^3. Stars are roughly spherical in shape and so the formula for density we will use is below.

$$density = \frac{Mass}{Volume}$$

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3}$$

<u>Example</u>

What is the density of a star with a mass of 3.00 x 10^{35} g and a radius of 2.00 x 10^{15} cm?

$$\rho = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3.00\,\text{x}\,10^{35}\,\text{g}}{\frac{4}{3}\pi\,(2.00\,\text{x}\,10^{15}\,\text{cm})^3} = \frac{3.00\,\text{x}\,10^{35}\,\text{g}}{3.35\,\text{x}\,10^{46}\,\text{cm}^3} = 8.96\,\text{x}\,10^{-12}\,\text{g/cm}^3$$

1.2 Schwarzschild Radius of a Black Hole

The Schwarzschild radius of a black hole is the distance from a black hole where the escape velocity is equal to the speed of light. Any distance closer than the Schwarzschild radius would require a speed *greater* than the speed of light to escape the gravitational pull of the black hole. Since nothing can move at speeds greater than the speed of light, anything that falls inside the Schwarzschild radius of a black hole is forever trapped by the hole. The Schwarzschild radius of a black hole is calculated from the formula below. G is the universal gravitational constant, M is the mass contained inside the black hole in kg, and c is the speed of light in m/s.

$$R_S = \frac{2\,G\,M}{c^2}$$

Example

What is the Schwarzschild radius of a black hole containing 8.00 x 10^{31} kg of mass? (note that this is 40 times the mass of the Sun)

$$\begin{aligned} R_S &= \frac{2\,G\,M}{c^2} = \frac{(2)(6.673\,\text{x}\,10^{-11})(8.00\,\text{x}\,10^{31})}{(3.00\,\text{x}\,10^{8})^2} \\ &= \frac{1.068\,\text{x}\,10^{22}}{9.00\,\text{x}\,10^{16}} \approx 119,000\,\text{m} \end{aligned}$$

It is *very* important that you use mass in kg and speed of light in m/s to get the correct answer in meters.